

Efficient Optimal Sizing And Allocation Of Capacitors In Radial Distribution Systems Using Drdlf And Differential Evolution

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Abstract— A distribution system is an interface between the bulk power system and the consumers. The radial distribution system is popular among these because of its low cost and simple design. The voltage instability in the power system is characterized by a monotonic voltage drop, which is slow at first and becomes abrupt after some time when the system is unable to meet the increasing power demand. Therefore to overcome these problems capacitors are used. The installation of the shunt capacitors on the radial distribution system is essential for power flow control, improving system stability, pf correction, voltage profile management and losses minimization. But the placement of the capacitors with appropriate size is always a challenge. Therefore for this purpose, in this paper along with Differential Evolution (DE) Algorithm, Dimension Reducing Distribution Load Flow (DRDLF) is used. This load flow identifies the location of the capacitors and the Differential Algorithm determines the size of the capacitors such that the cost of the energy loss and the capacitor to be minimum. In this problem the installation cost of the capacitors is also included. The above method is tested on IEEE 69 bus system and was found to be better compared to other methods like Genetic Algorithm and PSO.

Index Terms — Electrical Distribution Network, Optimal Capacitors Placement, Dimension reducing distribution load flow (DRDLF), Differential Evolution (DE) Algorithm.

I. INTRODUCTION

Capacitors are generally used for reactive power compensation in distribution systems. The purpose of capacitors is to minimize the power and energy losses and to maintain better voltage regulation for load buses and to improve system security. The amount of compensation provided with the capacitors that are placed in the distribution network depends upon the location, size and type of the capacitors placed in the system [1]. A lot of research has been made on the location of capacitors in the recent past [2], [3] without including the installation cost of the capacitors. All the approaches differ from each other by the way of their problem formulation and the problem solution method employed. Some of the early works could not take into account of capacitor cost. In some approaches the objective function considered was for control of voltage. In some of the techniques, only fixed capacitors are adopted and load

changes which are very vital in capacitor location was not considered. Other techniques have considered load changes only in three different levels. A few proposals were schemes for determining the optimal design and control of switched capacitors with non-simultaneous switching [4]. It is also very important to consider the problem solution methods employed to solve the capacitor placement problem, such as gradient search optimization, local variation method, optimization of equal area criteria method for fixed capacitors and dynamic programs [4], [5], [6]. Although these techniques have solved the problem, most of the early works used analytical methods with some kind of heuristics. In doing so, the problem formulation was oversimplified with certain assumptions, which was lacking generality. There is also a problem of local minimal in some of these methods. Furthermore, since the capacitor banks are non continuous variables, taking them as continuous compensation, by some authors, can cause very high inaccuracy with the obtained results. A differential evolution algorithm (DEA) is an evolutionary computation method that was originally introduced by Storn and Price in 1995 [18]. Furthermore, they developed DEA to be a reliable and versatile function optimizer that is also readily applicable to a wide range of optimization problems [19]. DEA uses rather greedy selection and less stochastic approach to solve optimization problems than other classical EAs. There are also a number of significant advantages when using DEA, which were summarized by Price in [20]. Most of the initial researches were conducted by the differential evolution algorithm inventors (Storn and Price) with several papers [18], [21], [22], [23] which explained the basis of differential evolution algorithm and how the optimization process is carried out. In this respect, it is very suitable to solve the capacitor placement or location problem. IEEE 69 bus distribution system is considered for case study. The test system is a 12.66 KV, 10 KVA, 69-bus radial distribution feeder consisting of one main branch and seven laterals containing different number of load buses. Buses 1 to 27 lie on the main branch. Bus #1 represents the substation feeding the distribution system.

II. DISTRIBUTION POWER FLOW

The distribution systems are characterized by their prevailing radial nature and high R/X ratio. This renders the load flow problem ill conditioned. So many methods [24-29] have been developed and tested ranging from sweep methods, to conic programming formulation. Early research indicated that standard load flow methods fail to converge for ill-conditioned test systems [30]. Esposito and Ramos [28] have proposed a radial load flow technique based on solving a system of equations in terms of new variables and using the Newton approach. The relationship between the complex branch powers and complex bus powers is derived as a non singular square matrix known as element incidence matrix.

The power flow equations are rewritten in terms of a new variable as linear recursive equations. The linear equations are solved to determine the bus voltages and branch currents in terms of new variable as complex numbers. The advantage of this algorithm is that it does not need any initial value and easier to develop the code since all the equations are expressed in matrix format. This proposed method could be applied to distribution systems having voltage-controlled buses also.

Notations

N-no of buses

I_{ij} -Branch current flowing through element ij

I_j -Bus current of node j

V_j -Bus voltage of node j

S_{ij} -Complex power flowing from node i to node j

S_{ji} -Complex power flowing received at node j from node i

S_j -Specified Bus power at bus j

Z_{ij} -Impedance of element ij

TL_{ij} -Transmission loss of element ij

The power flow method is summarized as follows:

1. For the first iteration transmission losses are initialized as zero for each element.
2. From the bus powers specified the branch powers are determined as per equation (1&2).

$$I_{branch} = K^{-1} I_{bus} \quad (1)$$

$$S_{bus} = K \left[S_{branch}^{sending} - TL_{branch} \right] \quad (2)$$

3. The variable R_{ij} is determined for each element using equation 3.

$$S_{ij} = P_{ij} + iQ_{ij} = R_{ij} Y_{ij}^* \quad (3)$$

4. The bus voltage, branch current and bus current are determined from R_{ij} .

$$V_j = V_i - \frac{R_{ij}^*}{V_i^*} \quad (4)$$

$$I_{ij} = \frac{R_{ij}^*}{V_i^*} Y_{ij} \quad (5)$$

5. The bus currents are determined

$$P_i + Q_i = \sum_{i \in k(i)} P_{ij} + iQ_{ij} = \sum_{i \in k(i)} V_i (V_i^* - V_j^*) Y_{ij}^* \quad (6)$$

And from (6) bus powers are calculated. Since the transmission losses are neglected in the first iteration there will be mismatch between the specified powers and calculated powers. The mismatch is a part of the transmission loss. TL_{ij}^r is the transmission loss part for 'i'th element for 'r'th iteration. Transmission loss of each element is the summation of the transmission loss portions of all previous iterations.

$$TL_{ij} = \sum^r TL_{ij} \quad (7)$$

'r' Where is the Iteration count

$$TL_{ij}^r = S_j^{spec} - {}^{r-1}V_j \cdot {}^{r-1}I_j^* \quad (8)$$

$$S_{ji} = S_{ij} - TL_{ij}$$

$$S_{branch}^{receiving} = S_{branch}^{sending} - TL_{loss} \quad (9)$$

$$\max(TL_{ij}^r) \leq \varepsilon$$

Treatment of voltage controlled buses

If power is fed from multiple ends of the radial system, other feeding buses except slack bus are treated as voltage controlled buses. The equation is as follows.

$$R_{ij} = V_i (V_i^* - V_j^*) \quad (10)$$

Equation 10 is modified for the jth voltage controlled bus.

$$real(S_{ij}) = P_{ij} = real(R_{ij} Y_{ij}^*) \quad (11)$$

$$R_{ij} = X_{ij} + iY_{ij} \quad (12)$$

$$P_{ij} = real((X_{ij} + iY_{ij})(G_{ij} + iB_{ij})) \quad (13)$$

$$P_{ij} = G_{ij} (|V_i|^2 - |V_i| |V_j| \cos(\phi_{12})) - B_{ij} |V_i| |V_j| \sin(\phi_{12}) \quad (14)$$

$$\frac{G_{ij} |V_i|^2 - P_{ij}}{|V_i| |V_j|} = G_{ij} \cos(\phi_{12}) - B_{ij} \sin(\phi_{12}) \quad (15)$$

The trigonometric equations are to be solved to get the phase angle of each PV bus j and the reactive power can be updated As

$$Q_{ij} = B_{ij} (|V_i|^2 - |V_i| |V_j| \cos(\phi_{12})) - G_{ij} |V_i| |V_j| \sin(\phi_{12}) \quad (16)$$

Then the same procedure described for the PQ buses is carried out till the convergence.

III. DIFFERENTIAL EVOLUTION

Differential evolution (DE) is a population-based stochastic optimization algorithm for real-valued optimization

problems. In DE each design variable is represented in the chromosome by a real number. The DE algorithm is simple and requires only three control parameters: weight factor (F), crossover rates (CR), and population size (NP). The initial population is randomly generated by uniformly distributed random numbers using the upper and lower limitation of each design variable. Then the objective function values of all the individuals of population are calculated to find out the best individual $x_{best,G}$ of current generation, where G is the index of generation. Three main steps of DE, mutation, crossover, and selection were performed sequentially and were repeated during the optimization cycle.

A. Mutation

For each individual vector $x_{i,G}$ in the population, mutation operation was used to generate mutated vectors in DE according to the following scheme equation:

$$v_{i,G+1} = x_{best,G} + F(x_{r1,G} - x_{r2,G}), i = 1, 2, 3, \dots, NP \quad (17)$$

In the Eq. 17, vector indices r1 and r2 are distinct and different population index and they are randomly selected. The selected two vectors, $x_{r1,G}$ and $x_{r2,G}$ are used as differential variation for mutation. The vector $x_{best,G}$ is the best solution of current generation. And $v_{i,G+1}$ is the best target vector and mutation vector of current generation. Weight factor F is the real value between 0 to 1 and it controls the amplification of the differential variation between the two random vectors. There are different mutation mechanisms available for DE, as shown Table. The individual vectors $x_{r1,G}$, $x_{r2,G}$, $x_{r3,G}$, $x_{r4,G}$, $x_{r5,G}$ are randomly selected from current generation and these random number are different from each other. So the population size must be greater than the number of randomly selected ion if choosing Rand/2/exp mechanism of DE mutation, the NP should be bigger than 5 to allow mutation.

TABLE I. THE MUTATION MECHANISM OF DE

Mechanism	Mathematical equation
Best / 1/ exp	$v_{i,G+1} = x_{best,G} + F(x_{r1,G} - x_{r2,G})$
Rand / 1/ exp	$v_{i,G+1} = x_{r3,G} + F(x_{r1,G} - x_{r2,G})$
Rand-to-Best	$v_{i,G+1} = x_{r1,G} + F(x_{r1,G} - x_{r2,G})$
Best / 2/ exp	$v_{i,G+1} = x_{best,G} + F(x_{r1,G} + x_{r2,G} - x_{r3,G} - x_{r4,G})$
Rand / 2/ exp	$v_{i,G+1} = x_{r1,G} + F(x_{r1,G} + x_{r2,G} - x_{r3,G} - x_{r4,G})$

B. Crossover

In the crossover operator, the trial vector $u_{i,G+1}$ is generated by choosing some arts of mutation vector, $v_{i,G+1}$ and other parts come from the target vector $x_{i,G}$. The crossover operator of DE is shown in Figure 1.

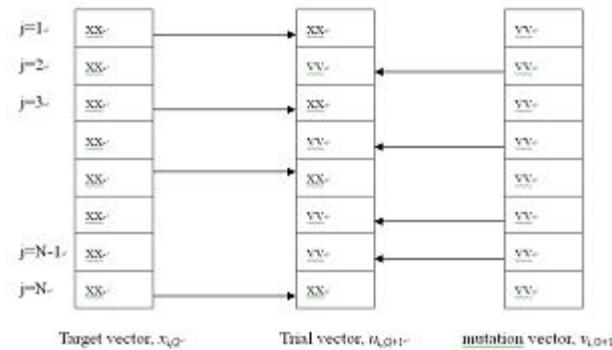


Figure 1 The Schematic diagram of crossover operation

Where Cr represents the crossover probability and j is the design variable component number. If random number R is larger than Cr value, the component of mutation vector, $v_{i,G+1}$ will be chose to the trial vector. Otherwise, the component of target vector is selected to the trial vectors. The mutation and crossover operators are used to diversify the search area of optimization problems.

C. Selection operator

After the mutation and crossover operator, all trial vectors $u_{i,G+1}$ have found. The trial vector $u_{i,G+1}$ are compared with the individual vector $x_{i,G}$ for selection into the next generation. The selection operator is listed in the following description:

$$x_{i,G+1} = u_{i,G+1}, \text{ if } f(u_{i,G+1}) > f(x_{i,G}) \\ x_{i,G+1} = x_{i,G}, \text{ if } f(u_{i,G+1}) \leq f(x_{i,G}), i = 1, 2, \dots, NP \quad (18)$$

If the objective function value of trial vector is better than the value of individual vector, the trial vector will be chosen as the new individual vector $x_{i,G+1}$ of next generation. On the contrary, the original individual vector $x_{i,G}$ will be kept as the individual vector $x_{i,G+1}$ in next generation. The optimization loop of DE runs iteratively until the stop criteria are met.

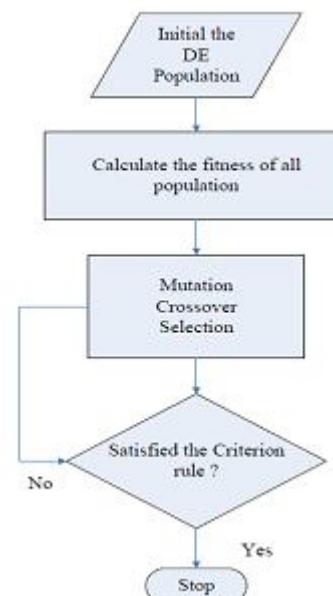


Figure 2. The flowchart of differential evolution

IV. IMPLEMENTATION OF DE

Algorithm to find capacitor sizes using DE:

The basic procedure of DE is summarized as follows.

Step 1: Randomly initialize population of individual for DE.

Step 2: Evaluate the objective values of all individuals, and determine the best individual.

Step 3: Perform mutation operation for each individual according to Eq. 17 in order to obtain each individual's corresponding mutant vector.

Step 4: Perform crossover operation between each individual and its corresponding mutant vector in order to obtain each individual's trial vector.

Step 5: Evaluate the objective values of the trial vectors.

Step 6: Perform selection operation between each individual and its corresponding trial Vector according to Eq.18 so as to generate the new individual for the next generation.

Step 7: Determine the best individual of the current new population with the best Objective value then updates best individual and its objective value.

Step 8: If a stopping criterion is met, then output gives its bests and its objective value.

Otherwise go back to step 3.

V. MATHEMATICAL FORMULATION

Objective Function for capacitor Sizing:

DE estimates the size of the capacitor to be installed by minimizing the following objective function.

$$S = K_e \sum_{j=1}^L T_j P_j + \sum_{i=1}^{ncap} (K_{cf} + K_c Q_{ci})$$

Where,

P_j = Power loss at j_{th} load level.

Q_{ci} = Reactive power injection from capacitor to node i

S = Savings in '\$'

T_j = Load Duration (8760 hrs)

$ncap$ = Number of Capacitor locations

L = Number of Load levels

K_e = Capacitor Energy Cost of Losses (0.06\$/kWh)

K_{cf} = Capacitor Installation Cost (1000\$)

K_c = Capacitor Marginal Cost (3\$/KVAR)

VI. TEST CASE CONSIDERED

Main Feeder Test System Specification:

IEEE Standard 12.66 KV, 69 Bus Systems

RADIAL FEEDER : 12.66 KV, 69 Bus Systems

LOAD : 1 P.U

NO OF LOAD LEVEL (L): 1

LOAD DURATION (T) : 8760

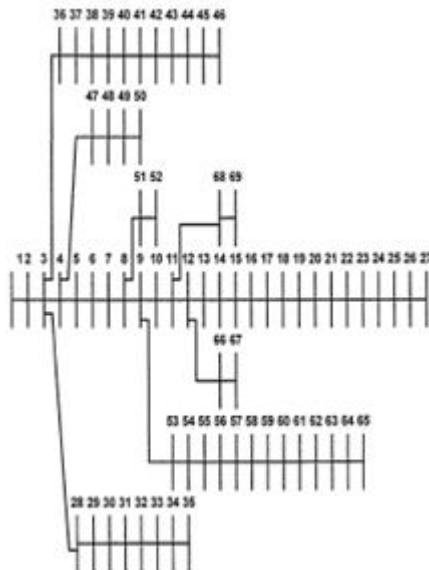


Figure 3: IEEE 69 bus system

VII. RESULTS AND DISCUSSIONS

Prior to capacitor installation, a load flow program based on complex power flow method is run to obtain the present system conditions. System conditions are shown in Table 2. The table specifies the minimum per-unit bus voltage, maximum per-unit bus voltage, real power losses in KW and the cost of energy losses during all load levels. It is clear from the table that the minimum bus voltages during the simulation are less than the pre-specified minimum allowable bus voltage. Therefore, capacitors shall be installed to provide the required voltage correction and to reduce the overall energy losses in the system. The proposed solution methodologies have been implemented in MATLAB 7.10.0. The solution algorithms based on DE algorithm and tested on IEEE 69 Bus System in Fig.3 which has been designed to find the optimal solution for this problem. In this case, only fixed type capacitors are installed in the system and all the loads are assumed to be linear. Computer programs have been written for these algorithms based on the respective procedures highlighted earlier. The parameters are defined as shown below:

Elapsed time: 92.38 Sec

$G_{max} = 800$; F: 0.8; CR: 0.8; NP: 100

Again, the parameters are set empirically by trial and error procedure. Parameters that have resulted in the best solution were chosen. A Differential Evolution based on steady-state replacement usually converges faster than the one designed based on generational replacement. Due to this, steady-state replacement method requires less number of generations before it converges to the optimal solution.

Table 2 shows the capacitor size obtained from DE method. From this table, we infer that total capacitor size in KVAR obtained by DE algorithm is quite less compared to the GA and PSO methods. From Table 2 it can be also observed that the results obtained using DE are compared and found to be better than the results obtained in the work under [17] & [33]

regarding net savings. The optimal placement and KVAR rating of shunt capacitor banks had been best determined for the studied distribution network using the proposed 'dimension reducing distribution load flow algorithm (DRDLFA)' and Differential Evolution.

Fig 4 shows the comparative results of average voltage profile before and after capacitor placement for both DE and GA methods. From Fig 4 it is observed that the voltage profile is improved after capacitor placement by 2.24% by DE compared to 2.38% by GA. From Fig 5, we infer that Net Savings increased in DE compared to GA & PSO. Although DE algorithm does not show a much increase in voltage profile, but provides a significant Savings.

TABLE II: SYSTEM CONDITIONS WITHOUT AND WITH CAPACITORS PLACEMENT FOR IEEE 69 BUS SYSTEM

Case	Before Capacitor	After Capacitor placement (proposed DE)	After Capacitor placement Using GA [33]	After Capacitor placement Using PSO [17]
Min bus voltage(pu)	0.9092	0.9296	0.9309	-
Max bus voltage(pu)	1	1	1	1
Real Power Loss(KW)	225.00	147.96	146.79	152.48
Optimum value of capacitor in KVAR	-	#16-200 #60-700 #61-500	#15-300 #60-600 #61-300 #63-300	#46- 241 #47- 365 #50-1015
Total KVAR placed	-	1400 KVAR	1500 KVAR	1621 KVAR
Energy loss cost (\$/Year)	1,18,260	77,767.77	77,152.82	80,143.48
Installation cost of the capacitors in (\$)	-	3000	4000	3000
Cost of the Capacitors (\$/Year)	-	4200	4500	4863
Net Savings (\$/Year)	-	33,292.23	32,607.18	30,253.52

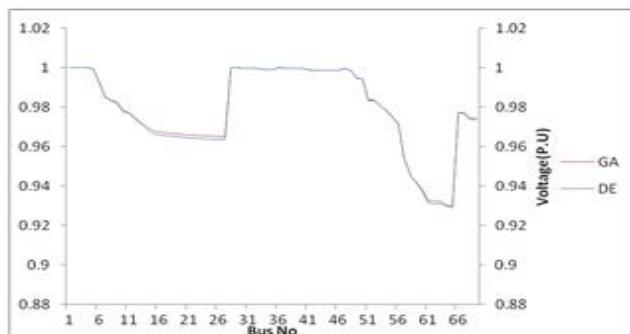


Figure 4: Comparison of Voltage Profiles of DE & GA Methods

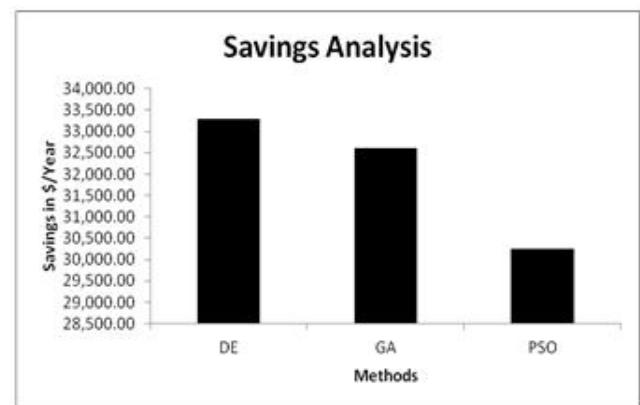


Fig 5: Savings Analysis Chart

VIII. CONCLUSION

It is concluded that savings in cost is maximum for DE than GA & PSO. Differential Evolution is the best method when compared to GA and PSO for optimal capacitor placement in Radial Distribution systems. This study presents DE method for Multi-objective programming to solve the IEEE 69 Bus Problem regarding Capacitor placement in the distribution system. The determined optimal location has reduced the system energy losses and consequently increased the net savings.

XI. FUTURE SCOPE

The advanced tools like Differential Evolution (DE) can be applied to same multi-objective problem of IEEE 69 Bus system for faster execution and the better results. The maintenance cost of the capacitor may be included in the cost function. However, the voltage profile can also be improved by considering the voltage constraints for each bus as a future Scope of the work.

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